# CHAOS IN MICROWAVE ANTENNA ARRAYS

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## Abstract

This paper considers a chaotic microwave transmitter consisting of a quasi-optical antenna array. When the coupling strength between array elements is too small to allow phase-locking, the antenna arrays exhibit low-dimensional chaos. These arrays show potential for use in inexpensive, high-power and high-speed wireless communication channels.

## Introduction

Quasi-optical oscillator arrays overcome many of the limitations of traditional power combining schemes. In these oscillator arrays, a set of antenna-loaded, single-device oscillators are fabricated in an array and coupled together through a transmission line network. This coupling permits synchronization through mutual injection locking. Although oscillator arrays are typically operated in the phase-locked regime, we propose that quasi-optical oscillator arrays are ideal platforms for chaotic communication systems. When the coupling strength between array elements is too small to allow phase locking, the antenna arrays exhibit low-dimensional chaos.

These chaotic arrays can be used as transmitters by employing a high speed microelectronic circuit to control the high power chaotic array. The free-running power stage is chaotic as the array 'switches' between various unstable periodic orbits; an infinite number of unstable periodic states typically coexist with any chaotic state. Since the chaotic state is arbitrarily close to any unstable periodic state, a small control perturbation can cause the (normally chaotic) signal from the power stage to follow an 'orbit' whose sequence represents the information to be communicated [1]. The strict separation of the power stage and the high speed electronics may allow for the fabrication of inexpensive, high speed wireless communication channels.

Control and modulation of a chaotic system requires thorough characterization of the system dynamics. In this paper, we obtain measures of the complexity and predictability of the system's dynamics. The Lyapunov exponents and the Kolmogorov-Sinai entropy are introduced and estimated. As a first step towards communications with chaotic antenna arrays, we employ the method of occasional proportional feedback (OPF) to stabilize the output of an oscillator array while it is in the chaotic regime.

## Coupled Oscillator Simulations

The theory of coupled microwave oscillators has been treated in previous work [2], leading to a set of differential equations for the time evolution of the amplitude and phase of each oscillator. For exploring chaotic behavior we are primarily interested in the case of weak coupling where the oscillators are unable to achieve a phase-locked state. In this limit the amplitude variations are insignificant, and the dynamics are governed by the phase equations;

$$\frac{d\theta_i}{dt} = \omega_i - \frac{\omega_i}{Q}\sum_{\substack{j=i-1 \\ j\neq i}}^{i+1} K_{ij}\sin\left(\Phi + \theta_i - \theta_j\right)$$

where $\theta_i$ is the phase of the $i$th oscillator output voltage and $\omega$ and $Q$ are the free-running oscillator frequency and quality factor, respectively. Note also that one of the



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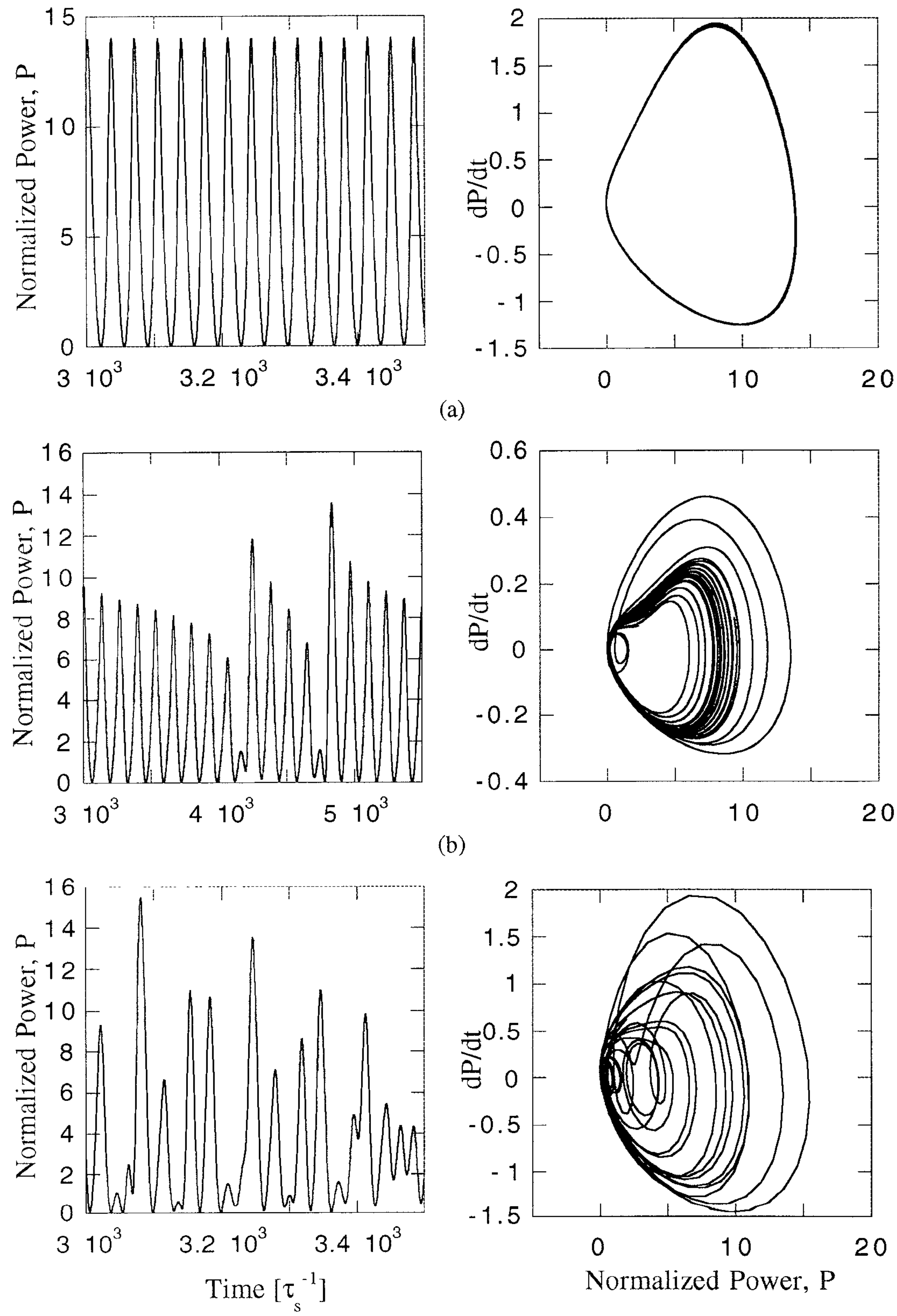

Figure 1. The far-field power radiated by a four element array. The power is plotted as a function of the time and as a function of its time derivative. Figure 1 (a), (b) and (c) correspond to coupling strengths of 0.025, 0.020, and 0.015, respectively.

phase variables is arbitrary and can be set to zero; an M-oscillator system has only M-1 degrees of freedom. The ability to predetermine the number of dynamic variables in the coupled oscillator systems makes this system an ideal candidate for chaos control.

The power transmitted by the antenna array is the natural variable for monitoring chaos in the coupled oscillators, and can be related to the parameters in Eq. 1 by a superposition of the field patterns for the individual antennas. In addition to the far-field power it is necessary that we monitor the signal locally for input to the control circuit. An obvious choice for local variable is the real part of the impedance into the antenna array.

In this paper, we use the coupling strength as the relevant control parameter. High-speed modulation of the coupling strength may be accomplished by introducing active elements in the coupling network of a transmission line coupled oscillator array. For example, integration of a FET into the coupling network would allow us to attenuate the coupling between neighboring elements simply by varying the gate bias.

The time series for the radiated power and the 'phase portrait' are shown in Fig. 1 for four values of the coupling strength. The time series is calculated for a four oscillator array with a frequency distribution of (9.988, 9.996, 10.004 and 10.012 GHz). For clarity, the 10 GHz carrier frequency has been removed from the time series data. Figure 2 shows a density plot of the numerical time series as a function of the coupling strength; the frequency of occurence for a given derivative of the radiated power is indicated by brightness. Three distinct regions can be identified in this figure. Coupling strengths greater than $\kappa = 0.064$ are the phase-locked regime. Below $\kappa = 0.020$ the dynamics appear chaotic. In the transition regime, the time series is quasi-periodic, but this periodicity is intermittently interrupted by large signal bursts. As the coupling strength is reduced towards the chaotic regime, the time between bursts becomes shorter until it is impossible to discern any clear periodicity. The transition to chaos does not follow the familiar period-doubling route to chaos. Similar intermittancy transitions to apparently chaotic behavior have been observed in fluid transport, heat convection and chemical reactions.

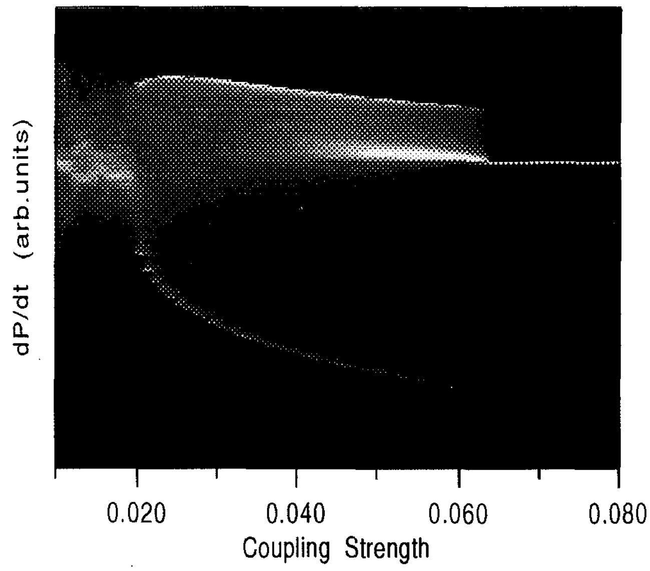


Figure 2. The global dynamics of the a four element coupled oscillator array.

### Analysis of Chaotic Time Series

Despite the apparent randomness, chaos refers to a specific, deterministic type of dynamics. There are rigid constraints on the predictability and complexity of a chaotic system's evolution.

The set of Lyapunov exponents $\lambda_i$ provides an intuitively appealing and yet powerful measure of sensitivity to initial conditions (SIC) and dissipation, both of which are required for a chaotic system. The set of $\lambda_i$ originates from a linear stability analysis, where the set of coupled differential equations (Eq. 1) are approximated by a first order Taylor expansion. In this approximation, all solutions are of the form $\exp(\lambda_i t)$, $i = 1..N$, where $N$ is the number of degrees of freedom. If one of the exponents is larger than 0, the distance between two initially nearby trajectories will increase exponentially with time in this direction; this exponential divergence is responsible for the SIC. The Lyapunov exponents [3] for the calculated time series are approximately +2, 0 and -6 (in units of the sampling time).

Kolmogorov-Sinai entropy is a second important measure of the system dynamics. The entropy represents the rate at which information is 'created' in the system. Consider a computer that tracks the state of dynamical systems and stores this information in a finite size memory. Two initial states of a chaotic system that are indistinguishable due to the limited precision of the memory become distinguishable as the trajectories diverge due to SIC. Therefore the amount of information needed to track a chaotic system is always increasing; the entropy is positive for a chaotic system; a periodic system will have an entropy of zero and a noise-driven system has an entropy that approaches infinity. The calculated estimate for the entropy in our oscillator arrays [4] is 2 (in units of the sampling time). Note that that the entropy is approximately equal to the positive Lyapunov exponent; this is not surprising given the relationship between the rate of information creation and the sensitivity to intial conditions.

Communication with these oscillator arrays requires that we control the output power from the oscillator arrays. Since the entropy was positive, the array is chaotic and we believe that there are an infinite number of unstable periodic orbits that are accessible by small perturbations on the coupling strength. In addition, the Lyapunov exponent gives us an estimate for the frequency of control perturbations needed to stabilize complex signal patterns. We have employed the method of occasional proportional feedback (OPF) [5] to modulate the coupling strength between oscillator elements. Modulation of the oscillator strength by a small fraction of its 'free-running' state has been used to stabilize a periodic oscillation of the radiated power.

### Coupled Oscillator Experiments

The complex dynamics of antenna arrays has also been investigated experimentally. Preliminary experiments with a three element and a four element oscillator array demonstrate complex aperiodic waveforms. The oscillators in the experiments consisted of MESFET loaded devices that were coupled by a transmission line network. These oscillators were used to drive four patch antennas. The center frequency of the radiation from the individual antennas and oscillators was approximately 8 GHz. Figure 3 shows the power detected by a horn antenna and digitizing oscilloscope along the broad side direction of the array. The fluctuations in the output power are clearly aperiodic. These preliminary experiments show great promise for chaos in high frequency coupled oscillator arrays.

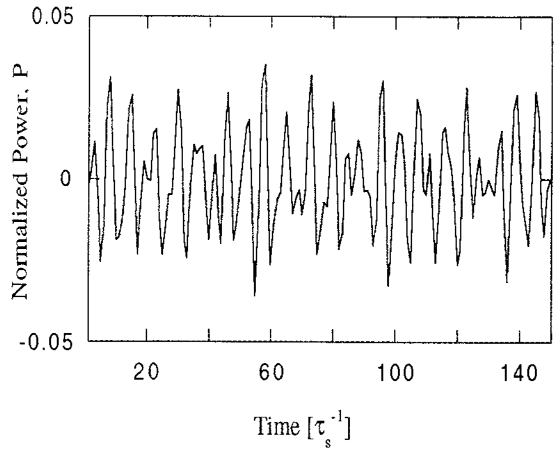


Figure 3. The experimental time series for a three element oscillator array with free-running frequency of 8 Ghz.

To summarize, we have characterized the dynamics of coupled antenna arrays with weak coupling between oscillator elements. The array dynamics have been demonstrated to exhibit chaos. In addition, controlling the output from these arrays by small control perturbations indicates the potential of such oscillator arrays for chaotic communications. The simplicity of the oscillator model is a good indication that these results apply to nearly all oscillator arrays regardless of load device our coupling geometry. Finally, the experiments with antenna arrays demonstrate the complex, aperiodic dynamics predicted by the theory.

**References**

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